Predicting the expected shortfall corresponding to value at risk forecasts produced by quantile models

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Abstract: The value at risk (VaR) is a tail quantile of the conditional distribution of a portfolio's return. It has become the standard measure of financial market risk, and hence has been used by banks over the last two decades for setting regulatory capital requirements. Although it is an intuitive measure of risk, the VaR provides no information regarding the possible exceedances beyond the VaR. A measure that addresses this, and which has a number of attractive theoretical properties, is the expected shortfall (ES), which is defined as the conditional expectation of exceedances beyond the VaR. Future regulatory frameworks are likely to put increased emphasis on the ES. Forecasts of the ES can be produced as a by-product of many methods used for VaR forecasting. Nonparametric approaches, such as kernel density estimation, and parametric approaches, such as GARCH modelling, deliver density forecasts from which both VaR and ES predictions can be obtained. Semiparametric approaches to VaR forecasting include those that model directly the conditional quantile for a chosen probability level using quantile regression. Directly modelling a quantile avoids the need for a distributional assumption, and allows the dynamics of a quantile to differ for each probability level. An example of such models is the conditional autoregressive VaR (CAViaR) class of models (see Engle and Manganelli 2004). In empirical studies, CAViaR models have performed well in comparison with parametric and nonparametric benchmarks. However, by focussing on a particular quantile, quantile regression models provide no information regarding the magnitude of the exceedances beyond the quantile, and hence no apparent way of producing ES forecasts. In this paper, we introduce a method for predicting the ES corresponding to the VaR forecasts produced by quantile regression models. The method enables the joint modelling of the VaR and ES, with parameters estimated using maximum likelihood based on an asymmetric Laplace density. This estimation approach has been shown to be equivalent to quantile regression (Koenker and Machedo 1999). We present an empirical study using daily stock indices.

Bibliography

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